CSE 595 Independent Study

Graph Theory

Week 4

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Chapter 3 Problem 1 (Nonseparable Graphs)



Looking at Theorem 3.2 in Chartrand [1] which states,

*A vertex in a graph is a cut-vertex of if and only if there are two vertices and distinct from such that lies on every path in .*

If a vertex were to lie further past , then the eccentricity .

Furthermore, if we were to delete from and there was another vertex with the same distance from then the shortest path does not include

Chapter 3 Problem 7 (Nonseparable Graphs)



1. If a graph has a vertex of degree 1, then this implies the vertex is a leaf. If this vertex is a leaf, then the graph has the same number of connected graphs as .
2. If , then order of . If this is true, then Theorem 3.3 [1] can be applied which states the following,

Let be a graph of order 3 of more. Then is nonseparable if and only if every two vertices of lie on a common cycle of .

If we follow that there is a cut-vertex of degree 2, then this implies that there are two subgraphs of connected by the cut-vertex. If the cut-vertex is removed, then we have the two subgraphs. If we remove either adjacent vertex of the cut-vertex, then the cut-vertex becomes a leaf for which ever graph the adjacent vertex belongs to that was not removed.

1. If we assume that for a graph of order , that the cut-vertex has a degree of , and all other vertices are of degree 1 which are connected to the cut-vertex. This allows for a graph with degree of with , which will always create subgraphs of order 1.

Chapter 3 Problem 17 (Nonseparable Graphs)

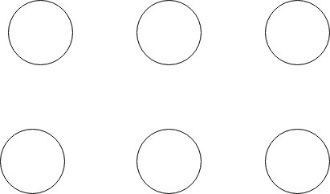


1. Chapter 3 Problem 1 can be looked in this exercise. A peripheral vertex in a graph has the largest eccentricity, meaning it has that largest distance from the vertex to another vertex in . Thus, if this is a peripheral vertex, then it follows the earlier above problem and a cut-vertex of cannot be a peripheral vertex of .
2. This is a false statement. Proving by counterexample, we can examine a graph with order 1. Although the one vertex in is a peripheral vertex, it is not a part of an end-block of *G*. In order to be an end-block, the subgraph of must contain one cut-vertex. A graph of order 1 does not contain any cut-vertex.

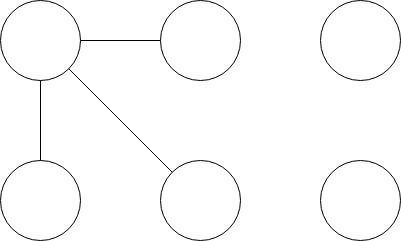
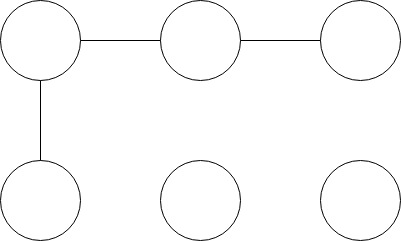
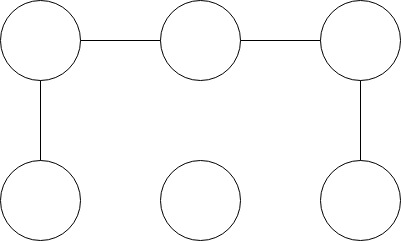
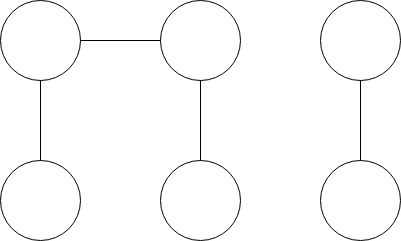
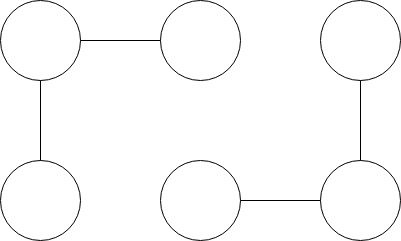
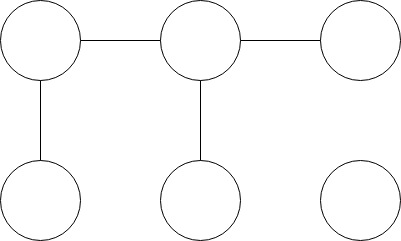
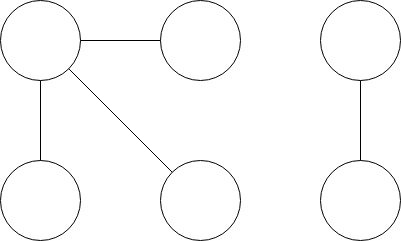
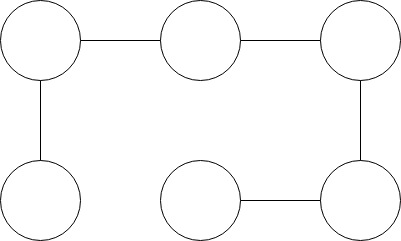
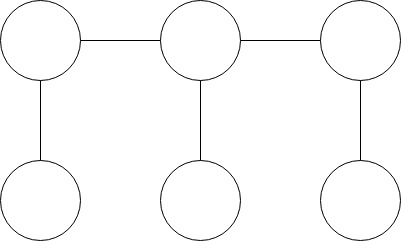
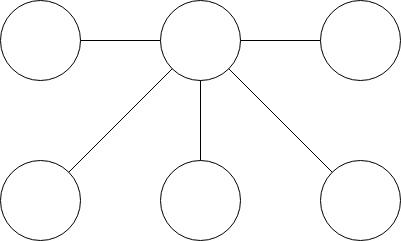
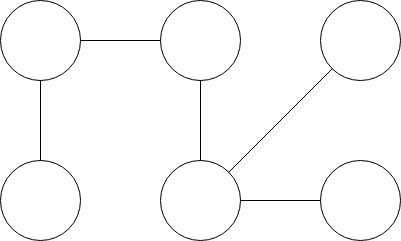
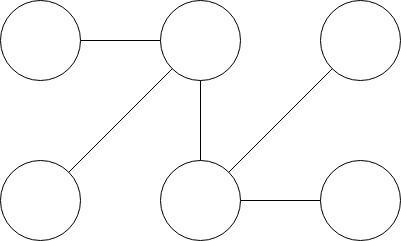
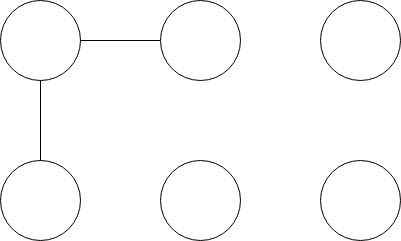
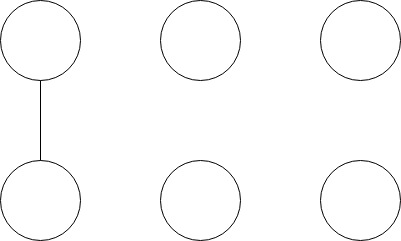
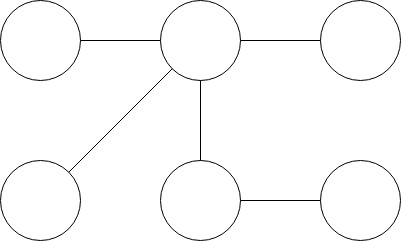
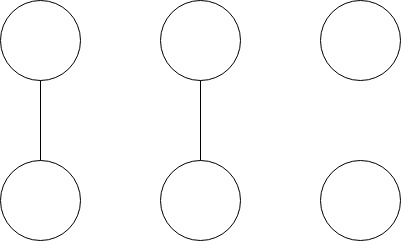
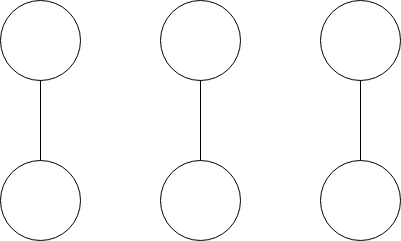
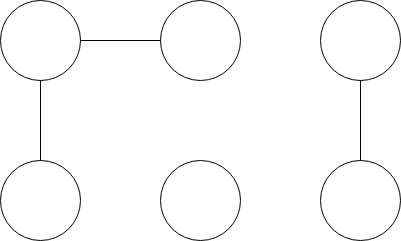
Chapter 3 Problem 19 (Introduction to Trees)

Following Theorem 3.10 [1] which states,

*An edge in a graph*  *is a bridge of*  *if and only if*  *lies on no cycle in*

Therefore, we know that does not lie on a cycle in . Since the order is of 3 or more, or . At least one of these vertices must therefore have one or more distinct vertices, and therefore is a cut-vertex of .Chapter 3 Problem 23 (Introduction to Trees)





Chapter 3 Problem 29 (Introduction to Trees)



Looking at Theorem 3.9 [1] which states,

*The center of every connected graph*  *lies in a single block of .*

Since the center of a graph ­ is a vertex where . There can only be a maximum of two vertices where is true in the case of a tree.

Two cases may be looked at then, a tree of order 1, and order 2

Order 1 – If the tree is of order 1, it is just a single vertex. Therefore, the graph could be a subgraph of a countably infinite number of supergraphs where cut-vertices have been removed.

Order 2 – If the tree is order 2, there are 2 vertices and one edge connecting them. Similarly to the case above, the graph may also be a subgraph of a countably infinite number of supergraphs where cut-vertices have been removed.

All other cases of trees will be separable graphs, who have a center block of either order 1 or order 2 .

Works cited

“Trees.” *Graphs & Digraphs*, by Gary Chartrand et al., CRC Press, 2016, pp. 57–69.